INFLUENCE OF THE VERTICAL SUSPENSION ON THE VIBRATION BEHAVIOR IN THE RAILWAY VEHICLES

MĂDĂLINA DUMITRIU[1]

Abstract: This paper brings into focus the influence of the vertical suspension damping on the vibration behavior in the railway vehicles. A complex model of the vehicle-track system is taken into account, which considers a series of factors that affect the level of the vertical vibrations. The movement equations are presented in an original manner, where the model of the vehicle-track system is described via 22 equations, divided into two decoupled systems defining the symmetrical and anti-symmetrical movements. The conclusion is that the velocity and the random nature of the rolling track irregularities are the ones to establish the strongest vibration modes. The best damping of the suspension leading to the minimizing in the vibration behavior is being shown.

Key words: railway vehicles, vertical vibrations, suspension, damping, random irregularity, frequency-domain analysis, geometric filtering

1. INTRODUCTION

While rolling, the railway vehicle is open to a constant behavior of vibrations whose main source is to be found in the phenomena of interaction with the track. Due to its construction, the track features as a whole, on the one hand, a number of deviations from the ideal geometrical shape and, on the other hand, defects of the rolling surfaces of the rails [1, 2]. These two above, along with the constructive discontinuities of the rail, make up for major reasons of the vehicle vibrations. At the wheel level, the defects in the rolling surface such as eccentricity, ovality, poligonal profile, corrugation, the wheel flat or flattening are also causes for the vibrations [3].

The vibrations in vehicles develop both vertically and horizontally but, due to its constructive symmetries (inertial, elastic and geometrical), the two types of vibrations are decoupled, thus being able to be studied separately.

Suspension plays an important role when it comes to the vibrations behavior in the vehicle, as it reduces the effect of the vertical, transversal and, sometimes, longitudinal shocks on the vehicle, thus providing both for meeting the requirements

[1] *Assistant Prof. PhD. Stud. Eng., Politehnica University of Bucharest, madalinadumitriu@yahoo.com*

derived from the rolling quality and compliance with the stipulations regarding the circulation safety.

Likewise, suspension is crucial for the vehicle's ability to help, from the perspective of the level of vibrations, with the comfort of passengers and transport of merchandise in good condition. Also, suspension contributes to reducing the stress under which the vehicle load-bearing structure is. While elastically assuming the shocks from the rolling track irregularities, by lowering the dynamic loads, suspension protects both the rolling device and the track [4].

The moment the velocity increases, there should be considered the issue of keeping the behavior of vibrations to a level inflicted by the homologation criteria of the railway vehicles from the view of rolling quality, rolling safety, comfort of passengers and track fatigue [5]. The solution would be a permanent improvement of the railway vehicles, and suspension is one of the priority items on the agenda.

The paper introduces the influence of the vertical suspension damping on the vibrations behavior at the vehicle carbody level, in three points defined as reference points in the vibrations level terminology.

For a correct evaluation of the railway vehicles dynamic behavior, mainly at high velocities, a complex model of the vehicle-track system is presented, which model considers the carbody elasticity, track elasticity and of the wheel-rail contact's, the influence of the conveyance system of the longitudinal stresses, as well as the effect of the geometrical filtering introduced by the distance between bogies [6 - 8].

Upon applying an original method based on the modal analysis, the model of the vehicle-track system is described in 22 movement equations, divided into two decoupled systems, one for the symmetrical and the other for the anti-symmetrical movements.

The paper also includes the results of the numerical simmulations concerning the response of the vehicle in a permanent vibration harmonic behavior, as well as the excitations represented by the random irregularities of the track. The vibration dominant modes are shown to be in a close relation with velocity and the random nature of the rolling track irregularities. Also, the possibility of establishing the best suspension damping is to be noticed, which reduces the vibrations behavior to a minimum.

2. MECHANICAL MODEL AND MOVEMENT EQUATIONS

The case here is of a four-axle vehicle, with two-stage suspension which travels at the constant speed *V* on a track with irregularities of longitudinal nivelment of a harmonic type. The mechanical model for the study of the vertical vibrations in the vehicle-track system is shown in figure 1.

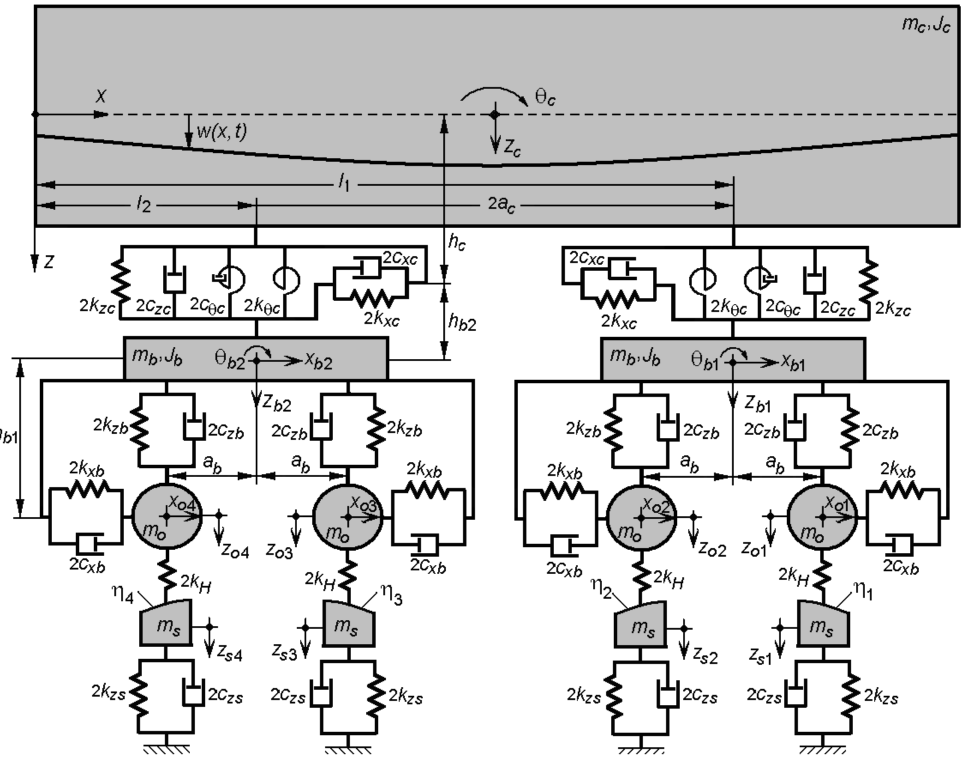

Fig. 1. The mechanical model of the vehicle-track system.

The model of the vehicle includes a body with parameters distributed for the carbody and a system of rigid bodies, respectively the wheelsets and the suspended masses of the two bogies. The carbody of the vehicle with length L and mass m_c is modelled by an Euler-Bernoulli beam of a constant section and a uniformly distributed mass, with the bending module EI, where E is the longitudinal module of elasticity and I is the inertia moment of the beam transversal section [9]. The structural analysis of the carbody structural.

The suspended masses of the bogies are considered rigid bodies on three degrees of freedom, with the following movements: bounce z_{bi}, forward x_{bi} and pitch θ_{bi}, where i = 1, 2. The mass of a bogie is m_b, and its inertia momentum $J_b = m_b i_b^2$, with i_b – the bogie gyration radius. The wheelsets of mass m_o have two degrees of freedom, thus generating a vertical movement of translation $z_{oj,(j+1)}$ and a longitudinal movement of translation $x_{oj,(j+1)}$, where $j = 2i$-1, and i = 1, 2, where each bogie is equipped with the wheelsets j and j+1.

Should we neglect the coupling effects between wheels derived from the propagation of the bending waves through rails, for the frequency range that is specific to the vehicle vertical vibrations, then a model equivalent with concentrated parameters will be adopted.

Against each wheelset in the bogie i, the track is represented by an oscillatory one- degree of freedom system that can move vertically, and the appropriate bounce is $z_{sj,(j+1)}$, where j = 2i-1 (for i = 1, 2). The track equivalent model has the mass m_s, stiffness k_{zs} and the damping coefficient c_{zs}.

The vehicle suspension, two levels on each bogie, are modelled by means of the Kelvin-Voigt systems. The primary suspension has two Kelvin-Voigt systems that operate on translation, vertically and longitudinally; on the other hand, the carbody suspension has three Kelvin-Voigt systems, two for translation (vertical and longitudinal) and one for rotation.

The elements of the Kelvin-Voigt system that take over the angular relative travelling between the carbody and bogie take into account the influence of the secondary suspension. The Kelvin-Voigt system on the longitudinal direction between the carbody and the bogie models the system of transmission of the longitudinal forces, located at distance h_c from the carbody neutral fiber and at distance h_{b2} from the center of gravity of the bogie's suspended mass. The longitudinal Kelvin-Voigt system located in the axle plan, at distance h_{b1} from the mass center of the bogie suspended mass, models the elastic steering of the wheelsets.

The features of the elastic connections of the secondary suspension, related to a bogie, are represented by the elastic constant values on vertical direction $2k_{zc}$, longitudinal $2k_{xc}$ and the pitch angular stiffness $2k_{\theta c}$, as well as the damping constant values $2c_{zc}$, $2c_{xc}$ and $2c_{\theta c}$. For the primary level of suspension, corresponding to an wheelset, the elastic constant values are noted with $2k_{zb}$ – on vertical direction and with $2k_{xb}$ – on longitudinal direction, while the damping ones are $2c_{zb}$, and $2c_{xb}$ respectively.

To calculate the frequency response, the track irregularities are considered to have a harmonic shape, with the wave length L and amplitude η_0

$$\eta(x) = \eta_0 \cos\frac{2\pi}{L}x, \tag{1}$$

where x is the coordinate along the track.

The size of the irregularities against each axle depends on their position. While considering the above equation to correspond to the track irregularity in the middle of the vehicle and that the position of the vehicle reported to the track is given in the relation $x = Vt$, then the defects against the axles are dephased by $2\pi(a_c \pm a_b)/lL$- against the front bogie and by $2\pi(-a_c \pm a_b)/L$ against of the axles of the rear bogie, where L is the defect wavelength, $2a_c$ is the distance between bogies and $2a_b$ stands for the bogie wheelbase. As a result, the deviations of irregularity will be

$$\eta_{1,2}(t) = \eta_0 \cos\omega\left(t + \frac{a_c \pm a_b}{V}\right);\ \eta_{3,4}(x) = \eta_0 \cos\omega\left(t - \frac{a_c \mp a_b}{V}\right), \tag{2}$$

where $\omega = 2\pi V/L$ is the angular frequency induced by the track excitation.

Further, once the mechanics laws are implemented, the movement equations of the vehicle-track system are as follows:

- for the carbody bending

$$EI\frac{\partial^4 w(x,t)}{\partial x^4}+\mu I\frac{\partial^5 w(x,t)}{\partial x^4 \partial t}+\rho_c\frac{\partial^2 w(x,t)}{\partial t^2}=\sum_{i=1}^{2}F_{zi}\delta(x-l_i)-\sum_{i=1}^{2}(M_i-h_c F_{xi})\frac{\mathrm{d}\delta(x-l_i)}{\mathrm{d}x} \quad (3)$$

where $\delta(.)$ is Dirac's delta function, and F_{xi}, F_{zi} and M_i are the forces and the moments derived from the secondary suspension of the bogie i.

The carbody movement equation, which includes partial derivatives, can be changed into equations with ordinary derivatives by implementing the method of modal analysis. For this purpose, the rigid and bending carbody modes are taken into account as

$$w(x,t)=z_c(t)+\left(x-\frac{L}{2}\right)\theta_c(t)+\sum_{n=2}^{\infty}X_n(x)T_n(t)\,, \quad (4)$$

where $z_c(t)$ and $\theta_c(t)$ are the carbody vibration rigid modes, namely the bounce and pitch, $T_n(t)$ is the time coordinate and $X_n(x)$ is the natural function of the mode n of vibration while bending

$$X_n(x)=\sin\beta_n x+\sinh\beta_n x-\frac{\sin\beta_n L-\sinh\beta_n L}{\cos\beta_n L-\cosh\beta_n L}(\cos\beta_n x+\cosh\beta_n x)\,; \quad (5)$$

where $$\beta_n=\sqrt{\omega_n^2 m/(EI)} \quad (6)$$

with ω_n as the natural angular frequency of the vibration mode n.

- for the bogie i bounce movement

$$m_b\ddot{z}_{bi}=\sum_{j=2i-1}^{2i}F_{zbj}-F_{zi}\,,\ \text{cu } i=1,2; \quad (7)$$

- for the bogie i pitch movement

$$J_b\ddot{\theta}_{bi}=a_b\sum_{j=2i-1}^{2i}(-1)^{j+1}F_{zbj}-h_{b1}\sum_{j=2i-1}^{2i}F_{xbj}-M_i-h_{b2}F_{xi}\,,\ \text{cu } i=1,2; \quad (8)$$

- for the bogie i rebound movement

$$m_b\ddot{x}_{bi}=\sum_{j=2i-1}^{2i}F_{xbj}-F_{xi}\,,\ \text{cu } i=1,2, \quad (9)$$

where F_{zbj} represents the forces due to the primary suspension and F_{xbj} the forces derived from the system of the axles elastic guidance.

- for the vertical movement of the axle j, and axle j +1 respectively

$$m_o \ddot{z}_{oj,(j+1)} = 2\Delta Q_{j,(j+1)} - F_{zbj,(j+1)}; \tag{10}$$

- for the longitudinal movement of the axle j, and for the axle j +1

$$m_o \ddot{x}_{oj,(j+1)} = -F_{xbj,(j+1)}, \text{ for } j = 2i\text{-}1 \text{ and } i = 1, 2. \tag{11}$$

To calculate the vertical dynamic forces, the hypothesis of the linear hertzian wheel-rail contact has been adopted

$$\Delta Q_{j,(j+1)} = -k_H [z_{oj,(j+1)} - z_{sj,(j+1)} - \eta_{j,(j+1)}], \text{ for } j = 2i\text{-}1 \text{ and } i = 1, 2 \tag{12}$$

where $\eta_{j,(j+1)}$, are the defects of the track longitudinal irregularity against the axle j, and j+1, and k_H – the rigidity of the wheel-rail contact.

-for the vertical movement of the rail against the axle j, and j +1

$$m_s \ddot{z}_{sj,(j+1)} = F_{zsj,(j+1)} - 2\Delta Q_{j,(j+1)}. \tag{13}$$

While considering only the first two natural modes of the carbody bending, symmetrical and anti-symmetrical, it results that the vibration of the vehicle-track system is described by a set of 22 equations, coupled with ordinary derivatives. Nevertheless, a correct choice of the coordinates and an appropriate interpretation of the system of equations will lead to its division into two independent systems of ten equations each, which systems describe the symmetrical and anti-symmetrical movements of the vehicle-track system and two decoupled movement equations. Thus, while writing

$$p_1^+ = z_c;\; p_1^- = \theta_c;\; p_2^+ = T_2;\; p_2^- = T_3;\; p_3^+ = \frac{1}{2}(z_{b1} + z_{b2});\; p_3^- = \frac{1}{2}(z_{b1} - z_{b2});$$

$$p_4^+ = \frac{1}{2}(\theta_{b1} - \theta_{b2});\; p_4^- = \frac{1}{2}(\theta_{b1} + \theta_{b2});\; p_5^+ = \frac{1}{2}(x_{b1} - x_{b2});\; p_5^- = \frac{1}{2}(x_{b1} + x_{b2});$$

$$p_6^+ = \frac{1}{4}(z_{o1} + z_{o2} + z_{o3} + z_{o4});\; p_6^- = \frac{1}{4}(z_{o1} + z_{o2} - z_{o3} - z_{o4});$$

$$p_7^+ = \frac{1}{4}(z_{o1} - z_{o2} - z_{o3} + z_{o4});\; p_7^- = \frac{1}{4}(z_{o1} - z_{o2} + z_{o3} - z_{o4});$$

$$p_8^+ = \frac{1}{4}(x_{o1} + x_{o2} - x_{o3} - x_{o4});\; p_8^- = \frac{1}{4}(x_{o1} + x_{o2} + x_{o3} + x_{o4});$$

$$p_9^+ = \frac{1}{4}(z_{s1} + z_{s2} + z_{s3} + z_{s4})\,; \quad p_9^- = \frac{1}{4}(z_{s1} + z_{s2} - z_{s3} - z_{s4})\,;$$

$$p_{10}^+ = \frac{1}{4}(z_{s1} - z_{s2} - z_{s3} + z_{s4})\,; \quad p_{10}^- = \frac{1}{4}(z_{s1} - z_{s2} + z_{s3} - z_{s4})\,,$$

the equations of the symmetrical movements are as follows

- the carbody symmetrical bounce

$$m_c \ddot{p}_1^+ + 4c_{zc}(\dot{p}_1^+ + \varepsilon^+ \dot{p}_2^+ - \dot{p}_3^+) + 4k_{zc}(p_1^+ + \varepsilon^+ p_2^+ - p_3^+) = 0\,; \tag{14}$$

- the carbody symmetrical bending

$$\begin{aligned} & m_{m2}\ddot{p}_2^+ + c_{m2}\dot{p}_2^+ + k_{m2}p_2^+ + 4c_{zc}\varepsilon^+(\dot{p}_1^+ + \varepsilon^+\dot{p}_2^+ - \dot{p}_3^+) + 4k_{zc}\varepsilon^+(p_1^+ + \varepsilon^+ p_2^+ - p_3^+) + \\ & + 4c_{xc}h_c\lambda^+[h_c\lambda^+\dot{p}_2^+ + h_{b2}\dot{p}_4^+ + \dot{p}_5^+] + 4k_{xc}h_c\lambda^+[h_c\lambda^+ p_2^+ + h_{b2}p_4^+ + p_5^+] + \\ & + 4c_{\theta c}\lambda^+(\lambda^+\dot{p}_2^+ - \dot{p}_4^+) + 4k_{\theta c}\lambda^+(\lambda^+ p_2^+ - p_4^+) = 0\,; \end{aligned} \tag{15}$$

- the symmetrical bounce of the bogies

$$\begin{aligned} & m_b\ddot{p}_3^+ + 4c_{zb}(\dot{p}_3^+ - \dot{p}_6^+) + 4k_{zb}(p_3^+ - p_6^+) + \\ & + 2c_{zc}(\dot{p}_3^+ - \dot{p}_1^+ - \varepsilon^+\dot{p}_2^+) + 2k_{zc}(p_3^+ - p_1^+ - \varepsilon^+ p_2^+) = 0\,; \end{aligned} \tag{16}$$

- the symmetrical pitch of the bogies

$$\begin{aligned} & J_b\ddot{p}_4^+ + 4c_{zb}a_b(a_b\dot{p}_4^+ - \dot{p}_7^+) + 4k_{zb}a_b(a_b p_4^+ - p_7^+) + \\ & + 4c_{xb}h_{b1}(h_{b1}\dot{p}_4^+ - \dot{p}_5^+ + \dot{p}_8^+) + 4k_{xb}h_{b1}(h_{b1}p_4^+ - p_5^+ + p_8^+) + \\ & + 2c_{xc}h_{b2}(h_c\lambda^+\dot{p}_2^+ + h_{b2}\dot{p}_4^+ + \dot{p}_5^+) + 2k_{xc}h_{b2}(h_c\lambda^+ p_2^+ + h_{b2}p_4^+ + p_5^+) + \\ & + 2c_{\theta c}(\dot{p}_4^+ - \lambda^+\dot{p}_2^+) + 2k_{\theta c}(p_4^+ - \lambda^+ p_2^+) = 0\,; \end{aligned} \tag{17}$$

- the symmetrical rebound of the bogies

$$\begin{aligned} & m_b\ddot{p}_5^+ + 4c_{xb}(\dot{p}_5^+ - h_{b1}\dot{p}_4^+ - \dot{p}_8^+) + 4k_{xb}(p_5^+ - h_{b1}p_4^+ - p_8^+) + \\ & + 2c_{xc}(h_c\lambda^+\dot{p}_2^+ + \dot{p}_5^+ + h_{b2}\dot{p}_4^+) + 2k_{xc}(h_c\lambda^+ p_2^+ + p_5^+ + h_{b2}p_4^+) = 0\,; \end{aligned} \tag{18}$$

- the vertical symmetrical movement of the axles

$$m_o\ddot{p}_6^+ + 2c_{zb}(\dot{p}_6^+ - \dot{p}_3^+) + 2k_{zb}(p_6^+ - p_3^+) + 2k_H(p_6^+ - p_9^+ - \eta_1^+) = 0\,; \tag{19}$$

$$m_o\ddot{p}_7^+ + 2c_{zb}(\dot{p}_7^+ - a_b\dot{p}_4^+) + 2k_{zb}(p_7^+ - a_b p_4^+) + 2k_H(p_7^+ - p_{10}^+ - \eta_2^+) = 0\,; \tag{20}$$

- the longitudinal symmetrical movement of the axles

$$m_o\ddot{p}_8^+ + 2c_{xb}(\dot{p}_8^+ + h_{b1}\dot{p}_4^+ - \dot{p}_5^+) + 2k_{xb}(p_8^+ + h_{b1}p_4^+ - p_5^+) = 0\,; \tag{21}$$

- the vertical symmetrical movement of the rails

$$m_s\ddot{p}_9^+ + 2c_{zs}\dot{p}_9^+ + 2k_{zs}p_9^+ + 2k_H(p_9^+ - p_6^+ + \eta_1^+) = 0\,; \tag{22}$$

$$m_s\ddot{p}_{10}^+ + 2c_{zs}\dot{p}_{10}^+ + 2k_{zs}p_{10}^+ + 2k_H(p_{10}^+ - p_7^+ + \eta_2^+) = 0\,; \tag{23}$$

and the equations of the anti-symmetrical movements

- the carbody anti-symmetrical pitch

$$\begin{aligned} &J_c\ddot{p}_1^- + 4c_{zc}a_c(a_c\dot{p}_1^- + \varepsilon^-\dot{p}_2^- - \dot{p}_3^-) + 4k_{zc}a_c(a_c p_1^- + \varepsilon^- p_2^- - p_3^-) + \\ &+ 4c_{xc}h_c[h_c(\dot{p}_1^- + \lambda^-\dot{p}_2^-) + h_{b2}\dot{p}_4^- + \dot{p}_5^-] + 4k_{xc}h_c[h_c(p_1^- + \lambda^- p_2^-) + h_{b2}p_4^- + p_5^-] + \\ &+ 4c_{\theta c}(\dot{p}_1^- + \lambda^-\dot{p}_2^- - \dot{p}_4^-) + 4k_{\theta c}(p_1^- + \lambda^- p_2^- - p_4^-) = 0\,; \end{aligned} \tag{24}$$

- the carbody anti-symmetrical bending

$$\begin{aligned} &m_{m3}\ddot{p}_2^- + c_{m3}\dot{q}_2^- + k_{m3}p_2^- + \\ &+ 4c_{zc}\varepsilon^-(a_c\dot{p}_1^- + \varepsilon^-\dot{p}_2^- - \dot{p}_3^-) + 4k_{cz}\varepsilon^-(a_c p_1^- + \varepsilon^- p_2^- - p_3^-) + \\ &+ 4c_{xc}h_c\lambda^-[h_c(\dot{p}_1^- + \lambda^-\dot{p}_2^-) + h_{b2}\dot{p}_4^- + \dot{p}_5^-] + \\ &+ 4k_{xc}h_c\lambda^-[h_c(p_1^- + \lambda^- p_2^-) + h_{b2}p_4^- + p_5^-] + \\ &+ 4c_{\theta c}\lambda^-(\dot{p}_1^- + \lambda^-\dot{p}_2^- - \dot{p}_4^-) + 4k_{\theta c}\lambda^-(p_1^- + \lambda^- p_2^- - p_4^-) = 0\,; \end{aligned} \tag{25}$$

- the anti-symmetrical bounce of the bogies

$$\begin{aligned} &m_b\ddot{p}_3^- + 4c_{zb}(\dot{p}_3^- - \dot{p}_6^-) + 4k_{zb}(p_3^- - p_6^-) + \\ &+ 2c_{zc}(\dot{p}_3^- - a_c\dot{p}_1^- - \varepsilon^-\dot{p}_2^-) + 2k_{zc}(p_3^- - a_c p_1^- - \varepsilon^- p_2^-) = 0\,; \end{aligned} \tag{26}$$

- the anti-symmetrical pitch of the bogies

$$\begin{aligned} &J_b\ddot{p}_4^- + 4c_{zb}a_b(a_b\dot{p}_4^- - \dot{p}_7^-) + 4k_{zb}a_b(a_b p_4^- - p_7^-) + \\ &+ 4c_{xb}h_{b1}(h_{b1}\dot{p}_4^- - \dot{p}_5^- + \dot{p}_8^-) + 4k_{xb}h_{b1}(h_{b1}p_4^- - p_5^- + p_8^-) + \\ &+ 2c_{xc}h_{b2}[h_c(\dot{p}_1^- + \lambda^-\dot{p}_2^-) + h_{b2}\dot{q}_4^- + \dot{p}_5^-] + 2k_{xc}h_{b2}[h_c(p_1^- + \lambda^- p_2^-) + h_{b2}p_4^- + p_5^-] + \\ &+ 2c_{\theta c}(\dot{p}_4^- - \dot{p}_1^- - \lambda^-\dot{p}_2^-) + 2k_{\theta c}(p_4^- - p_1^- - \lambda^- p_2^-) = 0\,; \end{aligned} \tag{27}$$

- the anti-symmetrical rebound of the bogies

$$\begin{aligned} & m_b \ddot{p}_5^- + 4c_{xb}(\dot{p}_5^- - h_{b1}\dot{p}_4^- - \dot{p}_8^-) + 4k_{xb}(p_5^- - h_{b1}p_4^- - p_8^-) + \\ & + 2c_{xc}[h_c(\dot{p}_1^- + \lambda^- \dot{p}_2^-) + h_{b2}\dot{p}_4^- + \dot{p}_5^-] + 2k_{xc}[h_c(p_1^- + \lambda^- p_2^-) + h_{b2}p_4^- + p_5^-] = 0\,; \end{aligned} \tag{28}$$

- the vertical anti-symmetrical movement of the axles

$$m_o \ddot{p}_6^- + 2c_{zb}(\dot{p}_6^- - \dot{p}_3^-) + 2k_{zb}(p_6^- - p_3^-) + 2k_H(p_6^- - p_9^- - \eta_1^-) = 0\,; \tag{29}$$

$$m_o \ddot{p}_7^- + 2c_{zb}(\dot{p}_7^- - a_b\dot{p}_4^-) + 2k_{zb}(p_7^- - a_b p_4^-) + 2k_H(p_7^- - p_{10}^- - \eta_2^-) = 0\,; \tag{30}$$

- the longitudinal anti-symmetrical movement of the axles

$$m_o \ddot{p}_8^- + 2c_{xb}(\dot{p}_8^- + h_{b1}\dot{p}_4^- - \dot{p}_5^-) + 2k_{zb}(p_8^- + h_{b1}p_4^- - p_5^-) = 0\,; \tag{31}$$

- the vertical anti-symmetrical movement of the rails

$$m_s \ddot{p}_9^- + 2c_{zs}\dot{p}_9^- + 2k_{zs}p_9^- + 2k_H(p_9^- - p_6^- + \eta_1^-) = 0\,; \tag{32}$$

$$m_s \ddot{p}_{10}^- + 2c_{zs}\dot{p}_{10}^- + 2k_{zs}p_{10}^- + 2k_H(p_{10}^- - p_7^- + \eta_2^-) = 0\,. \tag{33}$$

Similarly, the decoupled movement equations will be obtained, to describe the longitudinal movement of the axles in each bogie only

$$m_o(\ddot{x}_j - \ddot{x}_{j+1}) + 2c_{xb}(\dot{x}_j - \dot{x}_{j+1}) + 2k_{xb}(x_j - x_{j+1}) = 0, \text{ for } j = 2i\text{-}1 \text{ and } i = 1, 2. \tag{34}$$

The terms k_{mn}, c_{mn} and m_{mn}, (with n = 2, 3) included in the equations (15) and (25) stand for rigidity, damping and the modal mass, given by the expressions

$$k_{mn} = EI\int_0^L \left(\frac{\mathrm{d}^2 X_{nm}}{\mathrm{d}x^2}\right)^2 \mathrm{d}x\ ;\ c_{mn} = \mu I\int_0^L \left(\frac{\mathrm{d}^2 X_n}{\mathrm{d}x^2}\right)^2 \mathrm{d}x\,;\ m_{mn} = \rho_c \int_0^L X_n^2 \mathrm{d}x\,, \tag{35}$$

The movement equations contain two symmetrical modes of excitation

$$4\eta_1^+ = \eta_1 + \eta_2 + \eta_3 + \eta_4\,;\ \ 4\eta_1^- = \eta_1 + \eta_2 - \eta_3 - \eta_4 \tag{36}$$

and two anti-symmetrical modes of excitation

$$4\eta_2^- = \eta_1 - \eta_2 + \eta_3 - \eta_4\,;\ 4\eta_2^+ = \eta_1 - \eta_2 - \eta_3 + \eta_4\,. \tag{37}$$

due to the track irregularities against the axles.

3. BEHAVIOR OF RANDOM VIBRATIONS

As shown earlier, the study of the vehicle vertical vibrations involves the hypothesis that this vehicle runs on a track with random defects of a longitudinal nivelment. Also, it is assumed that the rolling track random defects are stationary in nature.

The spectral density power of the track irregularities can be approximated by a theoretical curve and the literature in review mentions various computation relations that generally concern the track quality. The recommended form by ORE [10] is featured below,

$$S(\Omega) = \frac{A\Omega_c^2}{(\Omega^2 + \Omega_r^2)(\Omega^2 + \Omega_c^2)}, \tag{38}$$

where Ω is the wave number, Ω_c = 0,8246 rad/m, Ω_r = 0,0206 rad/m, and $A = 4{,}032 \cdot 10^{-7}$ rad m or $A = 1{,}080 \cdot 10^{-6}$ rad m, depending on the track quality.

While noticing that the track defects become an excitation factor for the vehicle travelling at speed V, the spectral density power of the nivelment defects has to be expressed in dependence of the angle frequency $\omega = V\Omega$, as per the general relation

$$G(\omega) = \frac{S(\omega/V)}{V}. \tag{39}$$

The equations (38) and (39) will give

$$G(\omega) = \frac{A\Omega_c^2 V^3}{[\omega^2 + (V\Omega_c)^2][\omega^2 + (V\Omega_r)^2]}. \tag{40}$$

When starting with the vehicle frequency response and the range of the track defects, the spectral density power of the carbody vertical movement can be computed. Thus, for the response factor $\overline{H}_c(x,\omega)$ in a random point x along the carbody

$$\overline{H}_c(x,\omega) = \overline{H}_{z_c}(\omega) + \left(\frac{L}{2} - x\right)\overline{H}_{\theta_c}(\omega) + \sum_{n=2}^{3} X_n(x)\overline{H}_{T_n}(\omega), \tag{41}$$

where $\overline{H}_{z_c}(\omega)$, $\overline{H}_{\theta_c}(\omega)$, $\overline{H}_{T_i}(\omega)$ are the response factors corresponding to the rigid modes of vibration – bounce and pitch (z_c şi θ_c) and of the natural modes of symmetrical and anti-symmetrical bending ($T_{2,3}$), then the spectral density power of the carbody vertical movement will be

$$G_c(\omega, x) = G(\omega)\left|\overline{H}_c(\omega, x)\right|^2. \tag{42}$$

The equation (41) can be customized for various points along the carbody. Thus, in the middle of the carbody, the response factor is

$$\overline{H}_{cm}\left(\frac{L}{2},\omega\right)=\overline{H}_{z_c}(\omega)+X_2\left(\frac{L}{2}\right)\overline{H}_{T_2}(\omega), \tag{43}$$

and above the two bogies

$$\overline{H}_{cbi}(l_i,\omega)=\overline{H}_{z_c}(\omega)\pm a_c\overline{H}_{\theta_c}(\omega)+\sum_{n=2}^{3}X_n(l_i)\overline{H}_{T_n}(\omega)\text{, pentru } i = 1, 2. \tag{44}$$

The power spectral density of the carbody vertical acceleration are calculated by the relation

$$G_{ca}(\omega,x)=\omega^4 G(\omega)\left|\overline{H}_c(\omega,x)\right|^2, \tag{45}$$

valid for any point along the carbody.

4. NUMERICAL APPLICATION

This section features the results to the numerical simulations regarding the vehicle response in a steady-state harmonic behavior of vibrations, as well as the excitations represented by the random track irregularities. These simulations rely on the model and method shown in the previous section. The parameters of the numerical simulation have been properly defined for a passengers car equipped with bogies Y 32R, with the maximum velocity of 200 km/h.

In order to facilitate the analysis of vehicle vibrations behaviour, the damping ratio of each suspension level is introduced

$$\zeta_{zb,c}=\frac{4c_{zb,c}}{2\sqrt{4k_{zb,c}m_{b,c}}}. \tag{46}$$

In figure 2, the frequency response of the carbody at its centre (fig. 2, *a*) and against the two bogies (fig. 2, *b*) at speed of 200 km/h is shown, for the undamped case and for the reference values of the damping ratio (ζ_b = 0.22, ζ_c = 0.12).

At the centre of the carbody, its response is triggered by solely the symmetrical modes of vibration. For the undamped case, the peaks of the resonance frequencies can be noticed for the following symmetrical modes: the low bounce at 1.17 Hz and the high bounce at 6.61 Hz, the symmetrical carbody bending at 8.2 Hz, the bogie forward at 3.76 Hz and the bogie pitch at 9.63 Hz. Besides the resonance peaks, the anti-resonance frequencies are present, due to the geometric filtering derived from the distance between the bogies and the axle base. The introduction of damping lowers the level of vibrations against the resonance frequencies, but levels off, to a certain extent, the effect of the geometric filtering.

Above the bogies, the carbody vibration is made up of both the symmetrical vibration modes and anti-symmetrical, thus leading to an intensification of the vibration behaviour, under certain conditions. In other words, for the undamped case, the peaks of the resonance frequencies for the symmetrical movements, already mentioned, are accompanied by the ones for the anti-symmetrical movements, namely: the low frequency of the pitch at 1.46 Hz and the high frequency of the pitch at 9.61 Hz, the frequency of the anti-symmetrical bounce at 6.66 Hz, the anti-symmetrical carbody bending at 22.25 Hz and the bogie forward at 3.81 Hz. The lack of damping will make the carbody response above the two bogies symmetrical.

Above the bogies, the effect of the geometric filtering is less visible, since this place only the filtering effect of the axle base is present. The filtering due to the distance between the bogies does not operate in this case. When damping is considered, differences between the carbody response from one bogie to the other are quite significant. Such differences explain why the level of carbody vibration is not the same above the two bogies.

A final observation regards the fact that, when damping is introduced, the level of vibration for frequencies of up to circa 12 Hz is lower – mainly for the resonance areas – and significantly intensifies at high frequencies.

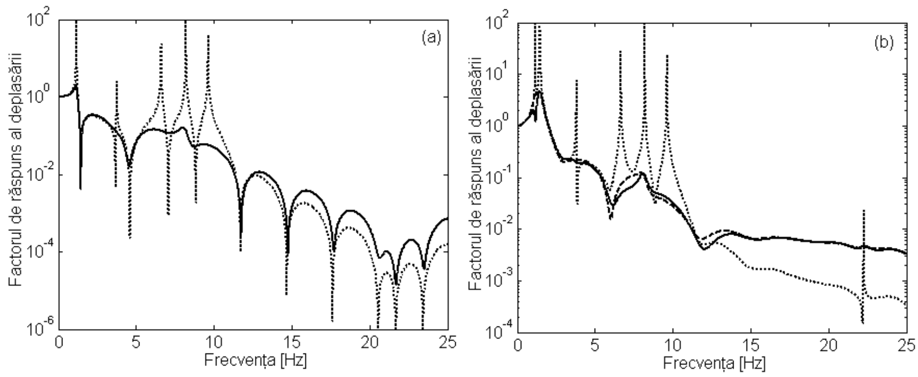

Fig. 2. Influence of damping on carbody response : (a) in the center , · · · ·, $\zeta_c = 0$, $\zeta_b = 0$; ——, $\zeta_c = 0.12$, $\zeta_b = 0.22$; (b) above the two bogies, · · · ·, $\zeta_c = 0$, $\zeta_b = 0$; ——, above the front bogie; − −, above the rear bogie, $\zeta_c = 0.12$, $\zeta_b = 0.22$.

The suspension damping plays an important role in terms of the carbody vibrations behaviour. Figure 3 shows the frequency response of the acceleration in the centre of carbody and against the two bogies for the reference speeds of 100 and 200 km/h, when considering the following values of the damping degrees: $\zeta_b = 0.22$, $\zeta_c = 0.12$ – the reference values and $\zeta_b = 0.44$, $\zeta_c = 0.24$, respectively.

The damping has opposite effects that depend on both the vibration modes and velocity. For examples, a higher damping will lower the carbody response at the low resonance bounce frequency. On the other hand, a higher damping will trigger an ampler response in the centre of the carbody within the bending resonance frequency range, which is noticed at 100 km/h; on the contrary, at 200 km/h, a higher damping reduces the vibration level. For high frequencies, a higher damping leads to intensification in the vibrations behaviour.

The diagrams in figure 3 also present the fact that, at the velocities there, the vehicle response is dominated by the carbody low bounce and by its symmetric bending.

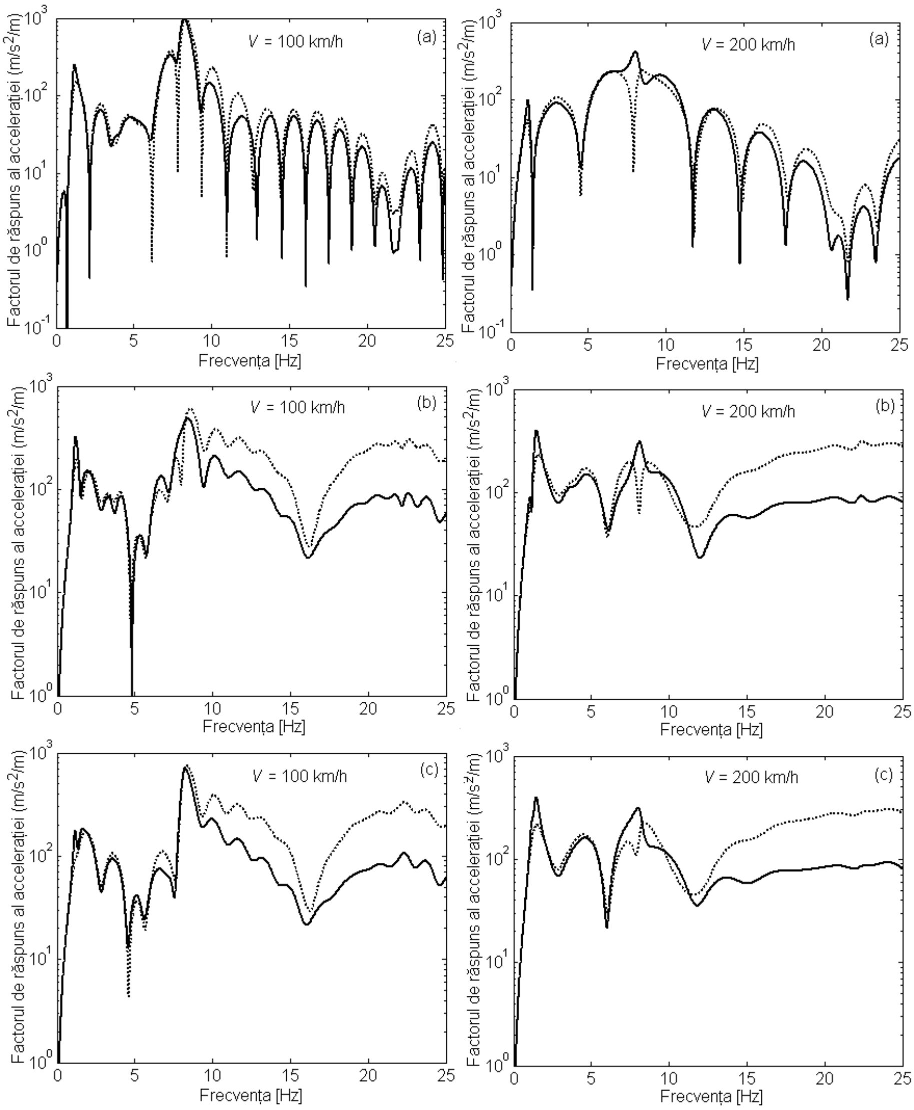

Fig. 3. Influence of suspension damping on the response factor of the carbody acceleration: (a) in the center; (b) above the front bogie; (c) above the rear bogie; —, ζ_b = 0,22, ζ_c = 0,12; · · · ·, ζ_b = 0,44, ζ_c = 0,24.

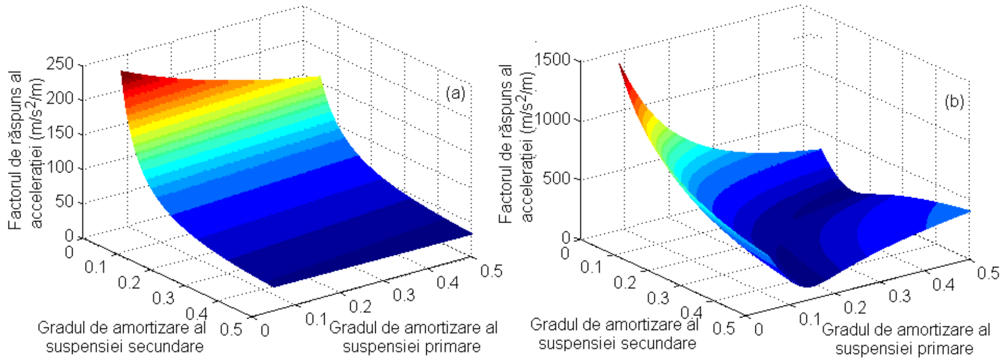

Fig. 4. Influence of the suspension damping on the response factor of carbody acceleration in the center: (a) at the low resonance bounce frequency (1.17 Hz); (b) at the resonance frequency of carbody symmetrical bending (8.2 Hz).

This is why it should be interesting to see in detail how the suspension damping influences the vehicle response for the two modes of vibrations (see figure 4). It can be noticed that, for a higher damping, the vibration level from the low bounce will continuously lower for the values being taken into account (fig 4, a). As for the influence of damping on the carbody vibration at the natural bending mode frequency (fig 4, b), it is quite interesting to see that for the small damping, the vibration level decreases along with the increase in damping, until it reaches a minimum value. Further, the level of vibration goes up with the increase of damping, due to the fact that the suspension dynamic stiffness is increasing. This aspect is the prerequisite of the existence of the best damping that minimizes the vehicle vibration at the velocities where the carbody bending dominates the vibrations behaviour.

The behavior of vibrations at the carbody level is established by both natural characteristics of the vehicle-track system and also the range of the track irregularities. In a nutshell, a clear image is obtained upon considering the two above characteristics. Figure 5 shows the power spectral density of the carbody in the middle and above the two bogies when vehicle is running on a high quality track (constant $A = 4{,}032\cdot10^{-7}$ rad m) at the velocities of 100 and 200 km/h.

It is interesting to notice that, for the above velocities, unlike the carbody response factor, (see fig. 3) where the dominant frequency is given by the symmetrical bending mode, the power spectral density of the acceleration is commanded by the low modes of the carbody vibration (bounce and pitch). This issue can be justified by the fact that the spectrum of the power spectral density of the track irregularities goes down along with the frequency.

In the middle of the carbody, as seen in fig 5, *a'*, a dominant value, is the low bounce. This statement is valid for a wide horizon of velocities – nevertheless, there is an interval of velocities focused on the value of 170 km/h where the vibration in the middle of the carbody is dominated by the carbody bending due to the geometrical filtering effect that, at these velocities, will fade out the influence of the low bounce.

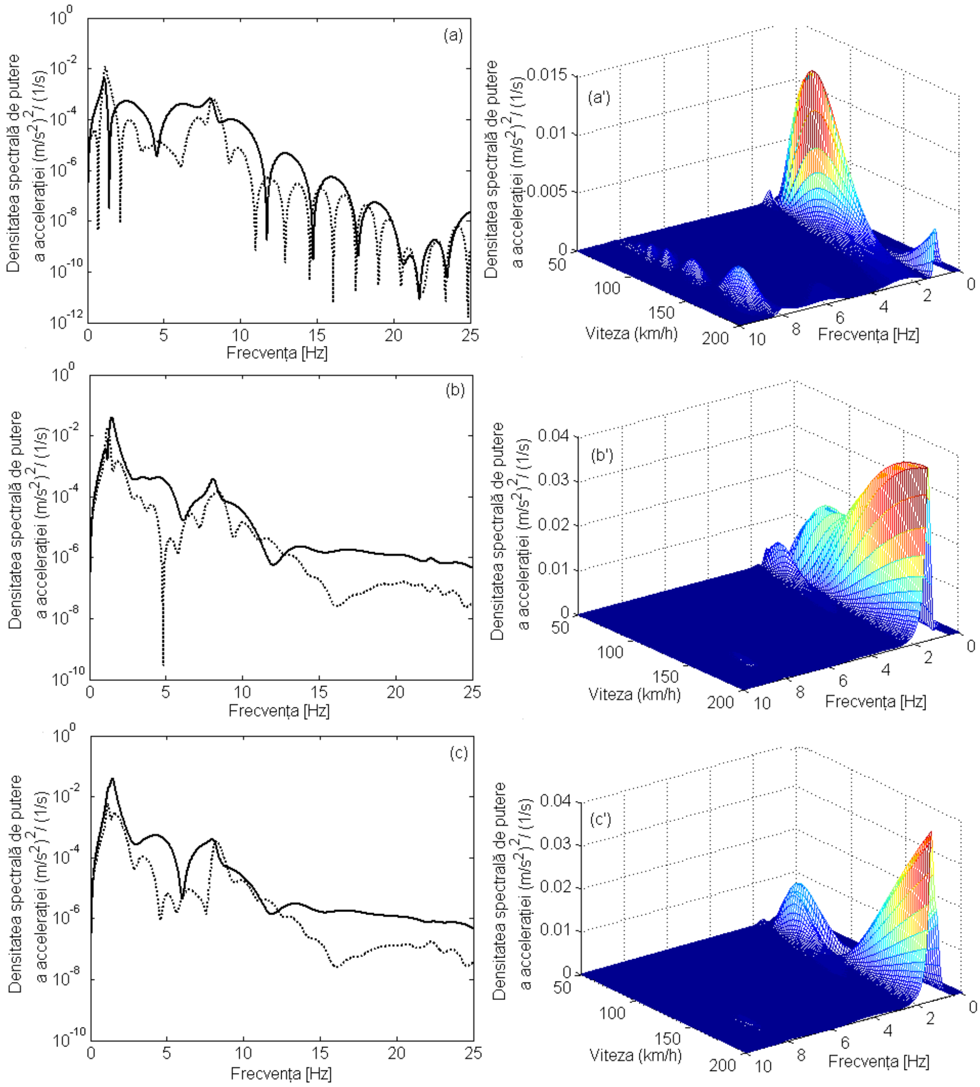

Fig. 5. Acceleration power spectral density:
(a) and (a') in the carbody middle; (b) and (b') above the front bogie;
(c) and (c') above the rear bogie; —, V = 100 km/h; · · · ·, V = 200 km/h.

Above the two bogies, the low bounce and the pitch dominate, with their frequencies being very close. The spectral density here is in the shape of a summit. On the other hand, the different nature of the vibration behavior above the two bogies can be noticed. Whereas the power spectral density goes up continuously with the velocity at the first bogie (there is no influence from the geometric filtering effect whatsoever), the velocity will be around 120 km/h for the second bogie, due to the geometric filtering effect from the distance between bogies.

5. CONCLUSIONS

The suspension of the railway vehicles plays an essential role in meeting the requirements brought about the homologation of the vehicles from the view of rolling quality, rolling safety, comfort of passengers and track fatigue.

This paper evaluates the influence of the damping of the suspension layers of a passenger vehicle upon its response to the random irregularities of the track. For this purpose, a complex discrete-continuous model of the vehicle-track system is being used. The movement equations are presented in an original manner, based on the modal analysis, in order to highlight the symmetrical and anti-symmetrical movements and the excitation modes.

It has been shown that damping will trigger the asymmetry of the vibration level on either side of the carbody center. On the other hand, damping reduces the vibration level in the field of low frequencies and leads to the vibration intensification in the range of high frequencies, of over 12 Hz. The existence of the best damping has been proven, which reduces the vibration level to a minimum. This issue has a strong practical meaning, since it provides a rational base to the improvement in designing the suspension for the railway vehicles. A connection has been made among damping, geometric filtering effect and the velocity, which will determine the vehicle dynamic performance.

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